STUDY OF KALMAN, EXTENDED KALMAN AND UNSCENTED KALMAN FILTER

(AN APPROACH TO DESIGN A POWER SYSTEM HARMONIC ESTIMATOR)

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By

MAMATA MADHUMITA
Roll No: 10602006

SOUMYA RANJAN AICH
Roll No: 10602012

Under the guidance of

ASST. PROF. SANKARSAN RAUTA

Department of Electrical Engineering
National Institute of Technology
Rourkela
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National Institute of Technology
Rourkela

CERTIFICATE

This is to certify that the thesis entitled, "**STUDY OF KALMAN, EXTENDED KALMAN AND UNSCENTED KALMAN FILTER (AN APPROACH TO DESIGN A POWER SYSTEM HARMONIC ESTIMATOR)**" submitted by Mamata Madhumita and Soumya Ranjan Aich in partial fulfillment of the requirements for the award of Bachelor of Technology Degree in Electrical Engineering at the National Institute of Technology, Rourkela (Deemed University) is an authentic work carried out by them under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other university / institute for the award of any Degree or Diploma.

Date:

Asst.Prof.Sankarsan Rauta

Dept. of Electrical Engineering

National Institute of Technology, Rourkela

Rourkela – 769008

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Mamata Madhumita
Roll no. : 10602006

Soumya Ranjan Aich
Roll no. : 10602012

Date:

Place: Rourkela

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LIST OF ABBREVIATIONS:

ASD	Adjustable Speed Drives
CBEMA	Computer and Business Equipment Manufacturer's Association
DFT	Discrete Fourier Transform
EKF	Extended Kalman Filter
FFT	Fast Fourier Transform
GRV	Gaussian Random Variable
Hz	Hertz
KF	Kalman Filter
KVA	Kilo Volt Ampere
LMS	Least Mean Square
MMSE	Minimum Mean-squared Error
MSE	Mean Square Error
p.u.	Per Unit
RMS	Root Mean Square
THD	Total Harmonic Distortion
THDF	Transformer Harmonic Derating Factor
UKF	Unscented Kalman Filter
UT	Unscented Transformation

LIST OF FIGURES

ABSTRACT

The accurate measurement of harmonics level is essential for designing harmonic filters, monitoring the stress to which the power system devices are subjected due to harmonics and specifying digital filtering techniques for phasor measurements for relaying. This project presents an integrated approach to design an optimal estimator of harmonic components of a power network in the presence of frequency variations. This has led to the study of Kalman, Extended Kalman and Unscented Kalman filter characteristics and a subsequent implementation of the study to design the optimal filter. We have employed the Extended Kalman filter and Unscented Kalman filter algorithms to estimate the power system voltage magnitude in the presence of random noise and distortions again taking into account the measurement noise. Kalman filter being an optimal estimator track the signal corrupted with noise and bearing harmonic distortion quite accurately. Adaptive tracking of harmonic components of a power system can easily be done using these algorithm. The proposed approaches are tested for only static signals. For a test signal both EKF and UKF algorithms are used and the results are compared.

CHAPTER – 1

<u>INTRODUCTION</u>

1.1 Introduction

1.INTRODUCTION

1.1 Introduction

Electric utilities are becoming more concerned about power system harmonics and voltage distortion. This increased concern is due to the increase in the application of power electronic devices in almost all kind of operation such as rectifiers and inverters used in motor drives. This results in increased injection in power system harmonics in the system. Again due to increase in application of shunt and series capacitors in the system, static var controllers at strategic locations for power factor correction there are high chances of increase potential for resonant conditions which can magnify the existing harmonic levels. The power system components continuously inject time varying harmonics in the system giving rise to non-stationary harmonic voltages and currents in the distribution system.

Al most all the real time functions are non-linear and all the systems can be represented as discrete time system to a great extent of accuracy using very small time stpes. Now the problem is to estimate the states of this discrete-time controlled process and the process is generally expressed with the help of linear stochastic difference equation. This estimation can be easily and accurately done by the Kalman Filter. But when the process and the measurement systems are non-linear EKF and UKF are implemented. A KF that linearizes about the current mean and covariance using any linearizing function is called extended KF. In this the partial derivatives of the process as well as measurement functions are used to compute estimates in the presence of non-linea functions.

CHAPTER – 2

POWER SYSTEM HARMONICS

2.1 Problems due to Voltage and Current Harmonics

2.2 Determination of presence of harmonic currents

2.3 Crest Factor

2.4 Measurement of Distorted Waveform

2.5 Characteristic and Non-characteristic Harmonics

2.6 Harmonic Estimation

2.POWER SYSTEM HARMONICS

Ideally, voltage and current waveforms are perfect sinusoids. However, because of the massive use of electronic and other non-linear loads the power system voltage and current waveforms become distorted. This deviation from a perfect sine wave can be represented by harmonics – sinusoidal components having a frequency that is an integral multiple of the fundamental frequency. To quantify the distortion, total harmonic distortion (THD) is used. This term expresses the distortion as a percentage of the fundamental of voltage and current waveforms.

2.1 Problems due to Voltage and Current Harmonics:

1. Current harmonics cause problem because they result in increased losses in the customer and utility power system components.[27] Transformers are very sensitive to this problem and may need to be derated as much as 50% capacity when feeding heavily non-linear loads with extremely distorted current waveform (current THD above 100%).[25]
2. Loads with severely distorted current waveforms also have a poor power factor which compels them to use excessive power system capacity causing overloading. Because of the highly distorted current Voltage source electronic adjustable speed drives (ASD) generally have a total power factor of approximately 65%. Using line side chokes the total power factor could be increased to approximately 85%.[27] The rate of rise and the peak value of the line current are checked using the chokes, drastically reducing the current THD. Current harmonics even distort the voltage waveform and cause voltage harmonics.[27]
3. Electric motors and capacitor banks also get affected along with sensitive electronic loads due to voltage distortion. In electric motors, negative sequence harmonics (i.e. 5^{th}, 11^{th}, 17^{th}…) have the potential to cause overheating as well as mechanical oscillations in the motor load system.[27]
4. Single phase non-linear loads generate odd harmonics (i.e. 3^{rd}, 5^{th}, 7^{th}, 9^{th}… etc).Some of the examples of such loads are personal computers, electronic ballasts and other electronic equipments. The 3^{rd} and other triplen harmonics are troublesome for single phase loads. Again the A-phase triplen harmonics, B-phase triplen harmonics and C-

phase triplen harmonics are all in the phase with each other and they add up on the neutral conductor of a 3-phase 4-wire system. This can overload the neutral if not sized to withstand this type of load.

5. The problem with capacitor banks is that with the increase in frequency the reactance of a capacitor bank decreases causing the bank to act as a trap or sink for higher harmonic currents. And this occurrence manifests in increased current, heating and dielectric stresses and this can even lead to capacitor bank failure.
6. Single–phase load harmonics vs. three-phase load harmonics additionally, triplen harmonics cause circulating currents on the delta winding of a delta-way transformer configuration. When current triplen harmonics on the neutral of a 3-phase 4-wire system reach the transformer, they are reflected to the delta connected primary where they circulate. The result is transformer heating similar to that produced by unbalanced 3-phase current. But the 3-phase non-linear loads like 3-phase ASDs, 3-phase DC drives, 3-phase rectifiers do not generate current triplens. On the otherhand they basically generate 5^{th} and 7^{th} current harmonics and a small amount of 11^{th}, 13^{th} and higher order harmonics.

2.2 Determination of presence of harmonic currents:

When non-linear loads are a considerable part of the total load in the facility(> 20%), there is chance of harmonic problem. So amount of current distortion produced by non-linear loads is calculated by calculating current THD.

1. Electronic ballasts come with current THD ranging from 60% to 100%. It's absolute necessary to avoid electronic ballasts with more than 20% current THD. Near to 100% current THD is produced by PWM ASDs. This can be easily brought down to less than half by installing cheap 3% impedance line-side reactors(chokes).
2. Again a measurement of the current in the neutral of a 3-phase 4-wire system gives knowledge about the presence of harmonics. If neutral current is found considerably higher than the value predicted from the imbalance in the 3-phase currents, it can be safely ssumed that harmonic currents are present in the system.
3. Another very important sign of presence of current harmonics include inexplicable higher than normal temperatures in the transformer, voltage distortion and high crest factor.

2.3 Crest Factor:

The crest factor of any waveform is the ratio of the peak value to the RMS value. In a perfect sine wave, the crest factor is 1.414. Crest factors different than 1.414 indicate distortion in the waveform. Typically distorted current waveforms have crest factors higher than 1.414 and distorted voltage waveforms have crest factors lower than 1.414. Distorted voltage waveforms with crest factors lower than 1.414 are called "flat top" voltage waveforms. The Computer and Business Equipment Manufactures Association (CBEMA) recommend a method for derating transformers based on the current crest factor. CBEMA defines the transformer harmonic derating factor(THDF) as the ratio of 1.414 to the transformer current crest factor. The derated KVA of the transformer would be the nominal KVA of the transformer multiplied by the THDF. This, method however is applied when the distortion in the current caused by single-phase, non-linear loads.[25]

2.4 Measurement of Distorted Waveform:

1. A digital oscilloscope is needed to measure the wave shape, THD and amplitude of each harmonic.
2. If only RMS value of the waveform is to be measured, a "True RMS" multimeter will suffice. 'True RMS ' is used because not all instruments give correct readings when measuring distorted waveforms.[25]
3. The majority of low-cost portable instruments are "average responding RMS calibrated". These instruments give correct readings for distortion free sine waves and will probably read low when the current waveform is distorted.[25]

2.5 Characteristic and Non-characteristic Harmonics:

The characteristic harmonics are harmonics of those order which are always present even under ideal operation – balanced AC voltages, symmetric three phase network and equidistant pulses. In the AC/DC converter the DC current is assumed to be constant. In this case, there are harmonics in AC current of the order $h = np + 1$, where p = pulse number, n is any integer.

The harmonics in the converter DC voltage are of the order $h = np$.

The harmonics of the order other than the characteristic harmonics are termed as non-characteristic. These are due to (i) imbalance in the operation of two bridges forming a 12 pulse converter(ii) firing angle errors (iii) unbalance and distortion in AC voltages and (iv) unequal transformer leakage impedances.[26]

Filters can be designed to eliminate the characteristic harmonics but it's very difficult fr complex analysis of the non-characteristic harmonics and therefore their elimination. So precautions are taken so that non-characteristic harmonics won't be generated.

2.6 Harmonic Estimation:

In order to provide the customers and electrical utilities a quality of power, it is imperative to know the harmonics parameters such as magnitude and phase. This is essential for designing filter for eliminating or reducing the effects of harmonics in a power system.[4]

Many algorithms have been proposed to evaluate the harmonics. To obtain the voltage and current frequency spectrum from discrete time samples, most frequency domain harmonic analysis algorithms are based on the discrete Fourier Transform (DFT) or on the fast Fourier transform (FFT)[4]. However, these two methods suffer three pitfalls, namely, aliasing, leakage and picket-fence effect [5] and [6] and [7]. Although other methods, including the proposed algorithm in this paper, suffer these three problems, and this is because of existing high-frequency components in the measured signal [5], however truncation of the sequence of sampled data, when only a fraction of the sequence of a cycle exists in the analyzed waveform, can boost leakage problem of DFT method. So the need for new algorithms that process the data, sample-by-sample, and not a window of data as in FFT and DFT, is of paramount importance.[4]

One of the methods is Kalman filter. Using fixed gain Kalman filter, a more robust algorithm was introduced by Dash and Sharaf [8] for estimating the magnitudes of sinusoids of known frequencies embedded in an unknown measurement noise which can be a mixture of both stochastic and signals. But this algorithm cannot track abrupt or dynamic changes of signal and its harmonics.[4]

In this paper we have studied the basics of Kalman Filter, Extended Kalman Filter and Unscented Kalman Filter. The characteristics and algorithms are thoroughly studied and a static Power system signal is simulated and results are compared.

CHAPTER – 3

THEORY OF KALMAN FILTER, EXETENDED KALMAN FILTER AND UNSCENTED KALMAN FILTER

3.1 Kalman Filter

3.1.1 The Process to be estimated

3.1.2 The computational origins of the filter

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3.2.1 Formulation

3.2.2 Predict and Update Equations

3.2.3 Flaws of EKF

3.3 Unscented Kalman Filter

3.3.1 Predict

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3.KALMAN FILTER, EXETENDED KALMAN FILTER AND UNSCENTED KALMAN FILTER

3.1 KALMAN FILTER:

The Kalman Filter is a mathematical method used to use observed values containing noise and other disturbances and produce values closer to true value and calculate value. This filter has many applications basically in the space and military technology.

The basic operation done by the KF is to generate estimates of the true and calculated values, first by predicting a value, then calculating the uncertainty of the above value and finding an weighted average of both the predicted and measured values. Most weight is given to the value with least uncertainty. The result obtained the method gives estimates more closer to true values.

3.1.1 The Process to be Estimated:

The KF addresses the basic problem of estimation of the state of a discrete-time controlled process that is governed by the linear stochastic difference equation.[3]

$$x_k = Ax_{k-1} + Bu_{k-1} + w_{k-1} \quad (3.1)$$

With a measurement:

$$z_k = Hx_k + v_k . \quad (3.2)$$

$$p(w) \sim N(0, Q) \quad (3.3)$$

$$p(v) \sim N(0, R) \quad (3.4)$$

The random variables in equations (3.1) and (3.2) represent the process and measurement noise respectively. They are assumed to be independent of each other or in other words they are uncorrlelated. The noise is assumed to be white and with normal probablity distributions. The *process noise covariance matrix Q* or *measurement noise covariance matrix R* may change with each time step or measurement, however we assume here they are constant matrices and in the differnce equation which relates the states at previous time step to the state at current step.[3]

3.1.2 The Computational Origins of the filter:

The $\hat{x}_k^- \in \Re^n$ is defined as the *a priori* state estimate at time step k when the process prior to step k is known, and the *a posteriori* state estimate at step k when the measurement is known. The *a priori* and *a posteriori* estimates errors can be defined as:

$$e_k^- \equiv x_k - \hat{x}_k^-, \text{ and}$$

$$e_k \equiv x_k - \hat{x}_k.$$

The *a priori* estimate error co-variance is then ,

$$P_k^- = E[e_k^- e_k^{-T}] \quad (3.5)$$

The *a posteriori* estimate error covariance is,

$$P_k = E[e_k e_k^T] \quad (3.6)$$

The next step involves finding an equation that computes an a posteriori state estimate as a linear combination of an a priori estimate and a weighted difference between an actual measurment and a measurement prediction.

$$\hat{x}_k = \hat{x}_k^- + K(z_k - H\hat{x}_k^-) \quad (3.7)$$

The kalman gain calculated from the equation:

$$K_k = P_k^- H^T (H P_k^- H^T + R)^{-1}$$

$$= \frac{P_k^- H^T}{H P_k^- H^T + R} \quad (3.8)$$

The difference $(z_k - H\hat{x}_k^-)$ is the measurement innovation or residual. We see that as the *R*, measurement error covariance approaches zero, the gain K weights the residual moe heavily.

3.1.3 Kalman Filtering Algorithm:

The Kalman Filter estimates a process by using a feedback control like form. The operation can be described as the process is estimated by the filter at some point of time and the feedback is obtained in the form of noisy measurements. The Kalman filter equations can be divided into two categories: *time update* equations and *measurement update* equations. To obtain the *a priori* estimates for the next time step the time update equations project forward (in time) the current state and error covariance estimates. The measurement update equations get the feedback to obtain an improved *a posteriori* estimate incorpoating a new measurement into the *a priori* estimate.

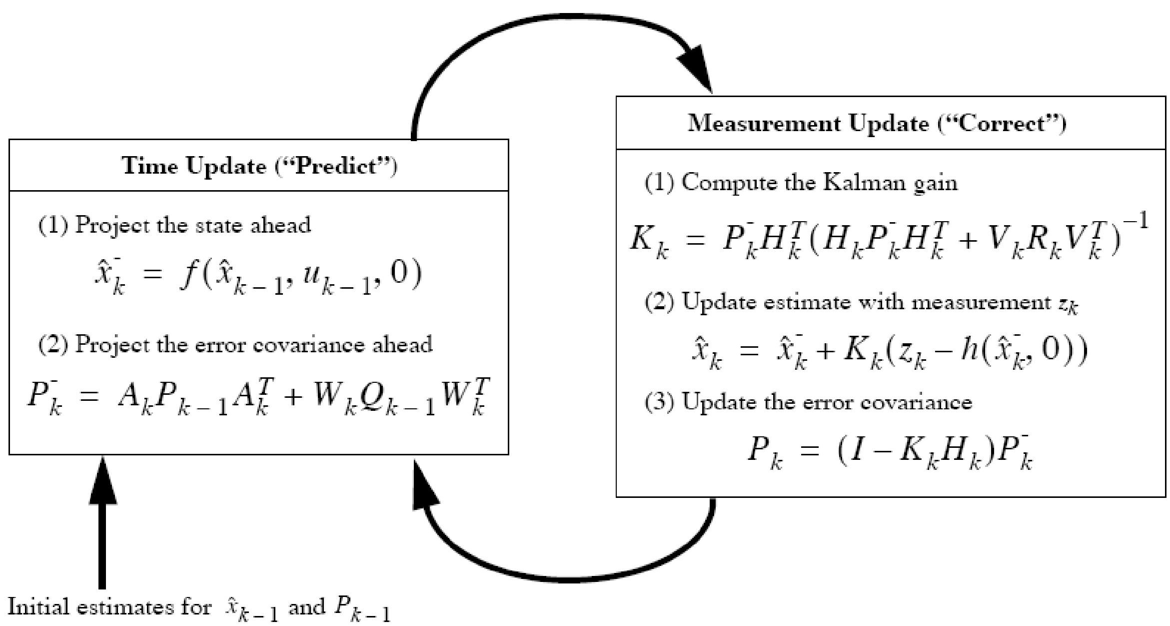


Fig 3.1: Kalman Filter Prediction Estimation Cycle

3.1.4 Underlying Dynamic System Model:

KF is based on linear and non-linear dynamical systems discretized in the time domain. A vector of real numbers represent the state of the system. At each discrete time increment, a new state is generated applying a linear operator, with some noise added.Then, the observed states are generated using another linear operator with some added noise usually called as the measurement noise.

To use the KF to get estimatations of the internal states of a process where only a sequence of noisy observations are known as inputs, the process is modeled in accordance with the state space representation of the Kalman filter. It means specifying the following matrices: the state-transition model, the observation model, the covariance of the process noise, the covariance of the observation noise; and sometimes the control-input model for each time-step, $_k$, F_k, H_k , Q_k, R_k, B_k, respectively as described further.

The KF model assumes the state at $(k-1)$ helps in measuring the true state at time k .

$$\mathbf{x}_k = \mathbf{F}_k \mathbf{x}_{k-1} + \mathbf{B}_k \mathbf{u}_k + \mathbf{w}_k$$

where

- $\mathbf{F}_k$ is the state transition state space model and it is applied to the previous state $\mathbf{x}_{k-1}$;
- $\mathbf{B}_k$ is the control-input state space model and it is applied to the control vector $\mathbf{u}_k$;
- $\mathbf{w}_k$ being the process noise and is drawn from a multivariate normal distribution with zero mean and covariance $\boldsymbol{Q}_k$.

$$\mathbf{w}_k \sim N(0, \mathbf{Q}_k)$$

An observation $\mathbf{z}_k$ of the true state $\mathbf{x}_k$ time k is made according to

$$\mathbf{z}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{v}_k$$

Here H_k is the observation state space model which helps in mapping the observed space from true space and v_k is the observation or measurement noise (Gaussian white noise) with zero mean and covariance R_k.

$$\mathbf{v}_k \sim N(0, \mathbf{R}_k)$$

Starting from the initial states to the noise vectors at each step are mutually independent.

A lot of real dynamical systems do not exactly fit this model as the KF mainly deals with linear systems and almost all real systems are non-linear. In fact, unmodelled dynamics can reduce the filter performance, though it is supposed to work finely with inputs which are unknown stochastic signals. The estimation algorithm can become unstable because the effect of unmodelled dynamics is dependent on the inputs. But the use of white Gaussian noise won't make the algorithm diverge and so in the project the noise used as input noise and measurement noise are Gaussian white noise.

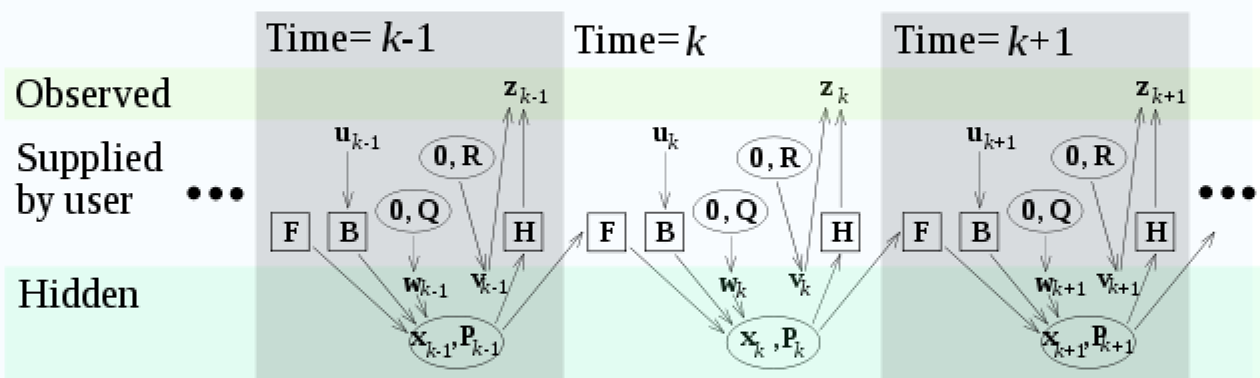


Fig 3.2: Model Underlying the Kalman Filter

3.1.5 Mathematical Formulation in steps:

The KF is a recursive estimator. Only the estimated state from the previous time step and the current measurement are required to compute the estimate for the current state.

The notation $\hat{\mathbf{x}}_{n|m}$ shows the estimate of $\mathbf{x}$ at time *n*, when observations till time *m* is obtained.

The two variables that can represent the filter:

- $\hat{\mathbf{x}}_{k|k}$, the *a posteriori* state estimate at time *k*
- $\mathbf{P}_{k|k}$, the *a posteriori* error covariance matrix (a measure of the estimated accuracy of the state estimate).

Predict

Predicted (*a priori*) state	$\hat{\mathbf{x}}_{k|k-1} = \mathbf{F}_k \hat{\mathbf{x}}_{k-1|k-1} + \mathbf{B}_k \mathbf{u}_k$
Predicted (*a priori*) estimate covariance	$\mathbf{P}_{k|k-1} = \mathbf{F}_k \mathbf{P}_{k-1|k-1} \mathbf{F}_k^{\mathrm{T}} + \mathbf{Q}_k$

Update

Innovation or measurement residual	$\tilde{\mathbf{y}}_k = \mathbf{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_{k|k-1}$
Innovation (or residual) covariance	$\mathbf{S}_k = \mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^{\mathrm{T}} + \mathbf{R}_k$
Optimal Kalman gain	$\mathbf{K}_k = \mathbf{P}_{k|k-1} \mathbf{H}_k^{\mathrm{T}} \mathbf{S}_k^{-1}$
Updated (*a posteriori*) state estimate	$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \tilde{\mathbf{y}}_k$
Updated (*a posteriori*) estimate covariance	$\mathbf{P}_{k|k} = (I - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_{k|k-1}$

3.2 EXTENDED KALMAN FILTER:

As we know the real systems that are inspiration for all these estimators like Kalman Filter are governed by non-linear functions. So we always need the advanced version of the Filters that are basically designed for linear filters. Similarly it is said that in estimation theory , the **extended Kalman filter** (**EKF**) is the nonlinear version of the Kalman filter. This non-linear filter linearizes about the current mean and covariance. At one time, the EKF might have been considered the standard in the theory of nonlinear state estimation navigation systems and GPS. However, as described below, with the introduction of the Unscented Kalman filter(UKF), the EKF might not enjoy the status of being the standard filter as the UKF is more robust and more accurate in its estimation of error.[1]

3.2.1 Formulation:

In the EKF, the state transition and observation state space models may not be linear functions of the state but might be many non-linear functions.

$$\mathbf{x}_k = f(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}) + \mathbf{w}_{k-1}$$
$$\mathbf{z}_k = h(\mathbf{x}_k) + \mathbf{v}_k$$

Where $\mathbf{w}_k$ and $\mathbf{v}_k$ are the process and observation noises which are both assumed to be zero mean multivariate Gaussian noise with covariance $\mathbf{Q}_k$ and $\mathbf{R}_k$ respectively.

The functions f and h use the previous estimate and help in computing the predicted state and again the predicted state is used to calculate the predicted measurement. However, f and h cannot be used to the covariance directly. So a matrix of partial derivatives (the Jacobian) computation is required.

At each time step with the help of current predicted states the Jacobian is calculated. These matrices are used in the KF equations. This process actually linearizes the non-linear function around the present estimate.[1]

3.2.2 Predict and Update Equations:

Predict

Predicted state	$\hat{\mathbf{x}}_{k\mid k-1} = f(\hat{\mathbf{x}}_{k-1\mid k-1}, \mathbf{u}_{k-1})$
Predicted estimate covariance	$\mathbf{P}_{k\mid k-1} = \mathbf{F}_{k-1}\mathbf{P}_{k-1\mid k-1}\mathbf{F}_{k-1}^{\top} + \mathbf{Q}_{k-1}$

Update

Innovation or measurement residual	$\tilde{\mathbf{y}}_k = \mathbf{z}_k - h(\hat{\mathbf{x}}_{k\mid k-1})$
Innovation (or residual) covariance	$\mathbf{S}_k = \mathbf{H}_k\mathbf{P}_{k\mid k-1}\mathbf{H}_k^{\top} + \mathbf{R}_k$
Optimal Kalman gain	$\mathbf{K}_k = \mathbf{P}_{k\mid k-1}\mathbf{H}_k^{\top}\mathbf{S}_k^{-1}$
Updated state estimate	$\hat{\mathbf{x}}_{k\mid k} = \hat{\mathbf{x}}_{k\mid k-1} + \mathbf{K}_k\tilde{\mathbf{y}}_k$
Updated estimate covariance	$\mathbf{P}_{k\mid k} = (I - \mathbf{K}_k\mathbf{H}_k)\mathbf{P}_{k\mid k-1}$

where the state transition and observation matrices are defined to be the following Jacobians

$$\mathbf{F}_{k-1} = \left.\frac{\partial f}{\partial \mathbf{x}}\right|_{\hat{\mathbf{x}}_{k-1\mid k-1}, \mathbf{u}_{k-1}}$$

$$\mathbf{H}_k = \left.\frac{\partial h}{\partial \mathbf{x}}\right|_{\hat{\mathbf{x}}_{k\mid k-1}}$$

3.2.3 Flaws of EKF:

The key to using an EKF is to be able to represent the system with a mathematical model. That is, the EKF designer needs to understand the system well enough to be able to describe its behavior with differential equation. In practice, this is often the most difficult part of implementing a Kalman filter. Another challenge in Kalman filtering is to be able to accurately model the noise in the system.

In the presence of non-linear functions the predicted states are approximated as the function of prior mean. The covariance are determined by linearizing the dynamic equations $(\mathbf{x}_{k+1} \approx \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{v}_k, \mathbf{y}_k \approx \mathbf{C}\mathbf{x}_\mathbf{k} + \mathbf{D}\mathbf{n}_k)$, and then the posterior covariance matrices are determined analytically for the linear system in EKF approach. In case of the EKF, a GRV is determined approximating the state distribution and then this GRV is propagated analytically through the ``first-order'' linearization of the nonlinear system. In this case the EKF can be credited with providing ``first-order'' approximations to the optimal terms such as optimal prediction, optimal gain. But these approximations, are not helpful always. Where the non-linearity value is more it can even introduce large errors in the true posterior mean as well as in the covariance of the transformed (Gaussian) random variable. This is not being an healthy approach to linearization might lead to sub-optimal performance and sometimes divergence (instability) of the filter. It is these ``flaws'' which will be amended in the next section using the UKF. [24]

3.3 UNSCENTED KALMAN FILTER (UKF):

When the state transition and observation state space models – the predict and update functions *f* and *h* are highly non-linear, the EKF cannot give up to the mark performance because the linearization of the underlying non-linear model propagates the covariance. A more deterministic sampling technique known as the unscented transform (UT) is used in the functioning of The *unscented Kalman filter (UKF)* . It determines a minimal set of sample points (called sigma points) around the mean. Then the non-linear functions are used to propagate these sigma points, from which the mean and covariance of the estimate are then recovered. This results in a filter which more accurately captures the true mean and covariance. This technique move out the requirement to explicitly calculate Jacobians, which is a bottleneck task for complex functions.

3.3.1 Predict

The UKF update can be used independently in the UKF prediction in co-ordination with a linear update.

The mean and covariance of the process noise is used in increasing the estimated state and covariance.

$$\mathbf{x}^a_{k-1|k-1} = [\hat{\mathbf{x}}^T_{k-1|k-1} \quad E[\mathbf{w}^T_k]\ ]^T$$

$$\mathbf{P}^a_{k-1|k-1} = \begin{bmatrix} \mathbf{P}_{k-1|k-1} & 0 \\ 0 & \mathbf{Q}_k \end{bmatrix}$$

The augmented state and covariance assist in derivation of a set of 2*L*+1 sigma points and L is the dimension of the augmented state.

$$\chi^0_{k-1|k-1} = \mathbf{x}^a_{k-1|k-1}$$

$$\chi^i_{k-1|k-1} = \mathbf{x}^a_{k-1|k-1} + \left(\sqrt{(L+\lambda)\mathbf{P}^a_{k-1|k-1}}\right)_i \quad i = 1..L$$

$$\chi^i_{k-1|k-1} = \mathbf{x}^a_{k-1|k-1} - \left(\sqrt{(L+\lambda)\mathbf{P}^a_{k-1|k-1}}\right)_{i-L} \; i = L+1,\ldots 2L$$

and

$$\left(\sqrt{(L+\lambda)\mathbf{P}^a_{k-1|k-1}}\right)_i$$

is the *i*th column of the matrix square root of

$$(L+\lambda)\mathbf{P}^a_{k-1|k-1}$$

The Cholesky decomposition metho should be used for the calculation of matrix square root. Because this method is numerically efficient and stable.

The transition function *f* helps in propagating the sigma points.

$$\chi^i_{k|k-1} = f(\chi^i_{k-1|k-1}) \quad i = 0..2L$$

The predicted state and covariance are produced by recombination of the weighted sigma points.

$$\hat{\mathbf{x}}_{k|k-1} = \sum_{i=0}^{2L} W_s^i \chi_{k|k-1}^i$$

$$\mathbf{P}_{k|k-1} = \sum_{i=0}^{2L} W_c^i \ [\chi_{k|k-1}^i - \hat{\mathbf{x}}_{k|k-1}][\chi_{k|k-1}^i - \hat{\mathbf{x}}_{k|k-1}]^T$$

And the weights for the state and covariance are given by:

$$W_s^0 = \frac{\lambda}{L+\lambda}$$

$$W_c^0 = \frac{\lambda}{L+\lambda} + (1 - \alpha^2 + \beta)$$

$$W_s^i = W_c^i = \frac{1}{2(L+\lambda)}$$

$$\lambda = \alpha^2(L+\kappa) - L$$

α and κ control the spread of the sigma points. β is related to the distribution of *x*. Normal values are $\alpha = 10^{-3}$, κ = 0 and β = 2. If the true distribution of *x* is Gaussian, β = 2 is optimal.[23]

3.3.2 Update

The same augmentation of the predicted state and covariance is done with the mean and covariance of the measurement noise.

$$\mathbf{x}_{k|k-1}^a = [\hat{\mathbf{x}}_{k|k-1}^T \quad E[\mathbf{v}_k^T] \ ]^T$$

$$\mathbf{P}_{k|k-1}^a = \begin{bmatrix} \mathbf{P}_{k|k-1} & 0 \\ 0 & \mathbf{R}_k \end{bmatrix}$$

Similarly a set of $2L + 1$ sigma points are derived.

$$\chi_{k|k-1}^0 = \mathbf{x}_{k|k-1}^a$$

$$\chi_{k|k-1}^{i} = \mathbf{x}_{k|k-1}^{a} + \left(\sqrt{(L+\lambda)\mathbf{P}_{k|k-1}^{a}}\right)_{i} \quad i = 1..L$$

$$\chi_{k|k-1}^{i} = \mathbf{x}_{k|k-1}^{a} - \left(\sqrt{(L+\lambda)\mathbf{P}_{k|k-1}^{a}}\right)_{i-L} \, i = L+1, \ldots 2L$$

If the UKF predictions to be used the sigma points are augmented as follows

$$\chi_{k|k-1} := [\chi_{k|k-1}^{T} \quad E[\mathbf{v}_{k}^{T}] \;]^{T} \pm \sqrt{(L+\lambda)\mathbf{R}_{k}^{a}}$$

where

$$\mathbf{R}_{k}^{a} = \left[\begin{array}{cc} 0 & 0 \\ 0 & \mathbf{R}_{k} \end{array} \right]$$

The observation function *h* projects the sigma points.

$$\gamma_{k}^{i} = h(\chi_{k|k-1}^{i}) \quad i = 0..2L$$

The predicted measurement and predicted measurement covariance are generated recombining the weighted sigma points.

$$\hat{\mathbf{z}}_{k} = \sum_{i=0}^{2L} W_{s}^{i} \gamma_{k}^{i}$$

$$\mathbf{P}_{z_k z_k} = \sum_{i=0}^{2L} W_{c}^{i} \; [\gamma_{k}^{i} - \hat{\mathbf{z}}_{k}][\gamma_{k}^{i} - \hat{\mathbf{z}}_{k}]^{T}$$

The state-measurement cross-covariance matrix,

$$\mathbf{P}_{x_k z_k} = \sum_{i=0}^{2L} W_{c}^{i} \; [\chi_{k|k-1}^{i} - \hat{\mathbf{x}}_{k|k-1}][\gamma_{k}^{i} - \hat{\mathbf{z}}_{k}]^{T}$$

Helps in computing the UKF Kalman gain.

$$K_k = \mathbf{P}_{x_k z_k}\mathbf{P}_{z_k z_k}^{-1}$$

In case of the KF, the updated state is the predicted state and the innovation weighted by the Kalman gain,

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + K_k(\mathbf{z}_k - \hat{\mathbf{z}}_k)$$

the updated covariance = the predicted covariance - the predicted measurement covariance, weighted by the Kalman gain.

$$\mathbf{P}_{k|k} = \mathbf{P}_{k|k-1} - K_k\mathbf{P}_{z_k z_k}K_k^T$$

CHAPTER – 4

COMPARISON OF KALMAN FILTER ESTIMATION APPROACHES

4.1 General and Linear State-Space Model

4.2 Kalman Filter

4.3 Extended Kalman Filter

4.4 Unscented Kalman Filter

4.COMPARISON OF KALMAN FILTER ESTIMATION APPROACHES:

4.1 General and Linear State Space Model:

The most general form of state space model is the non-linear model. The models are basically consist of two function *f* and *h* which govern the state propagation and measurements respectively. **w** and **v** are the process and measurement noises respectively, **u** is the process input and k is the discrete time.[2]

$$\mathbf{x}_{k+1} = f(\mathbf{x}_k, \mathbf{u}_k, \mathbf{w}_k)$$
$$\mathbf{z}_k = h(\mathbf{x}_k, \mathbf{v}_k)$$

This is the actual model where as the linear state-space model is the model where the functions *f* and *h* are both linear in state and input. The function then can be expressed using matrices F, B and H, reducing state propagations to linear algebra casing easier calculation and analysis. The model equations can be written as:

$$\mathbf{x}_{k+1} = F_k\mathbf{x}_k + B_k\mathbf{u}_k + \mathbf{w}_k$$
$$\mathbf{z}_k = H_k\mathbf{x}_k + \mathbf{v}_k$$

4.2 Kalman Filter:

Certain constraints on the process model can make the estimation problem easier. The constraints in case of Kalman filter are both the functions *f* and *h* are to be linear, noise terms **w** and **v** uncorrelated, Gaussian and white with zero mean. The mathematical formulation is given in section 3.1.5. The model being linear and input being Gaussian we have the knowledge that the output will also be Gaussian. The state and output pdf will therefore always be normally distributed and the knowledge of mean and covariance will suffice.[2] .Estimation using Kalman filter is easier as it incorporates almost all linear calculation except a matrix inversion.

4.3 Extended Kalman Filter:

Almost all real life process are non-linear and needed to be linearized before they can be estimated by means of a KF. By calculating the Jacobian of *f* and *h* around the estimated state, this problem of non-linearity is solved by EKF. The calculation of Jacobian yields a trajectory of the model function centered around the state. The mathematical formulation is given in the section 3.2.2.

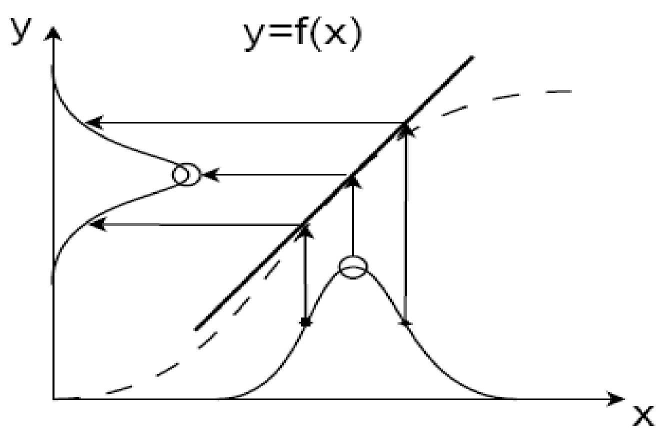


Fig-4.1: EKF linearizing a non-linear function around the mean of a Gaussian distribution

4.4 Unscented Kalman Filter:

Propagation of GRV through non-linear functions cause trouble many times and by applying a new technique called the *unscented transform* it can be solved. In stead of linearizing a non-linear function it uses 2N+1 sigma points for N states and then propagates these points through the actual non-linear function, eliminating linearization altogether. The points are chosen such that their mean, covariance as well as other higher order moments also match the GVR. These propagated points help in recalculating the mean and covariance yielding more accurate results compared to ordinary function linearization. The underlying idea is to approximate the probability distribution instead of the function. This strategy helps in decrement in computational complexities at the same time increasing estimation accuracy, gaining faster and more accurate results.

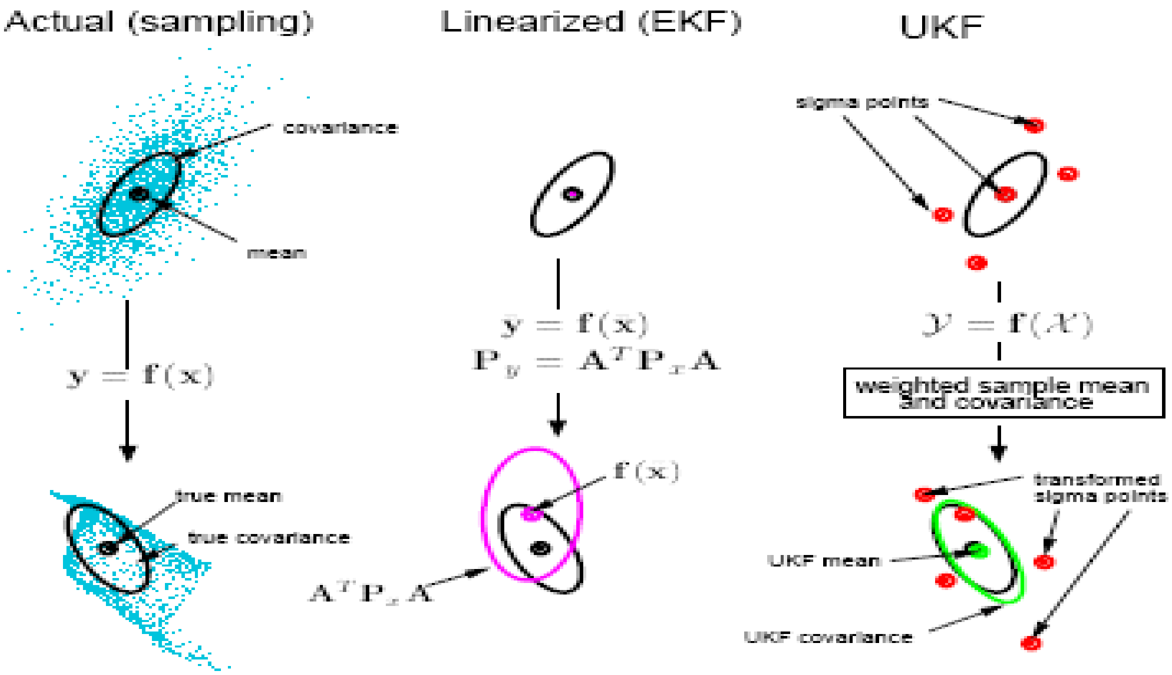


Fig-4.2: Example of mean and covariance propagation

The unscented transform approach provides another advantage of treating noise in a nonlinear fashion to account for non-Gaussian or non-additive noises. For doing so firstly noise is propagated through the functions by first augmenting the state vector including the noise sources. This technique was first introduced by Julier and later developed by Merwe. Sigma point samples are then selected from the augmented state $x_{a,}$ which includes the noise values. This technique results in the accuracy of process and measurement noise capture with same accuracy as that of the state, which in turn increases the accuracy for non-additive noise sources.

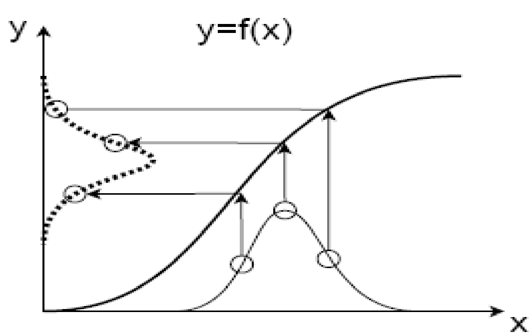


Fig-4.3: UKF propagating sigma points from a Gaussian distribution through a non-linear function

CHAPTER – 5

ESTIMATTION OF POWER SYSTEM HARMONICS USING KALMAN FILTER METHODOLOGY AND SIMULATION RESULTS

5.1 Estimation of Power System Harmonics using Kalman Filter methodology

5.2 EKF and UKF Simulation Results

5.1 ESTIMATTION OF POWER SYSTEM HARMONICS USING KALMAN FILTER METHODOLOGY:

Firstly a static signal as given below is modeled with the help of EKF and UKF algorithms.

$$x_k = (\sin x_{k)}^2 + \exp(x_k) + w_k$$

The test signal with these non-linearities and the Gaussian noise is used as the process and estimation of the amplitude is carried out with the help of EKF and UKF methods. The estimated and the original signal and a comparison between them is given in the following section in the form of Fig 5.1 and Fig 5.3. In case of EKF the mean square error is calculated for the estimated signal and given in Fig 5.2.

Consider a signal consisting of M sinusoids which can be modeled as given below.

$$y_k = \sum_{i=1}^{M} A_{ik} cos(\omega_i t_k + \phi_i) + v_k, \qquad k = 1, 2, 3, \ldots, N$$

Where A_{ik}, w_i , Φ_i , t_k and v_k are amplitude, frequency and phase of the i- th sinusoid respectively and t_k is the k-th sample of the sampling time and v_k is a zero mean Gaussian white noise. In this paper a signal like this has been used but keeping aplitude 1p.u. and frequency 50Hz and phases 0^o and generating process and measurement noises with the help of random number generator available in MATLAB "**randn**". The amplitude and frequency estimation has been done of the signal starting from fundamental frequency signal to 3rd, 5th and 7th harmonic signal. The results are shown in the next section from Fig 5.4 to Fig 5.11. Both the original and estimated voltage amplitude value and frequency values have been compared.

5.2 EXTENDED KALMAN FILTER AND UNSCENTED KALMAN FILTER SIMULATION RESULTS:

- Extended Kalman Filter Simulation Output
- Mean Square Error in Extended Kalman Filter
- Unscented Kalman Filter Output
- Amplitude and Frequency Estimation of a Test Signal using EKF
    - Fundamental frequency and amplitude estimation
    - 3^{rd} harmonic frequency and amplitude estimation
    - 5^{th} harmonic frequency and amplitude estimation
    - 7^{th} harmonic frequency and amplitude estimation

EXTENDED KALMAN FILTER OUTPUT

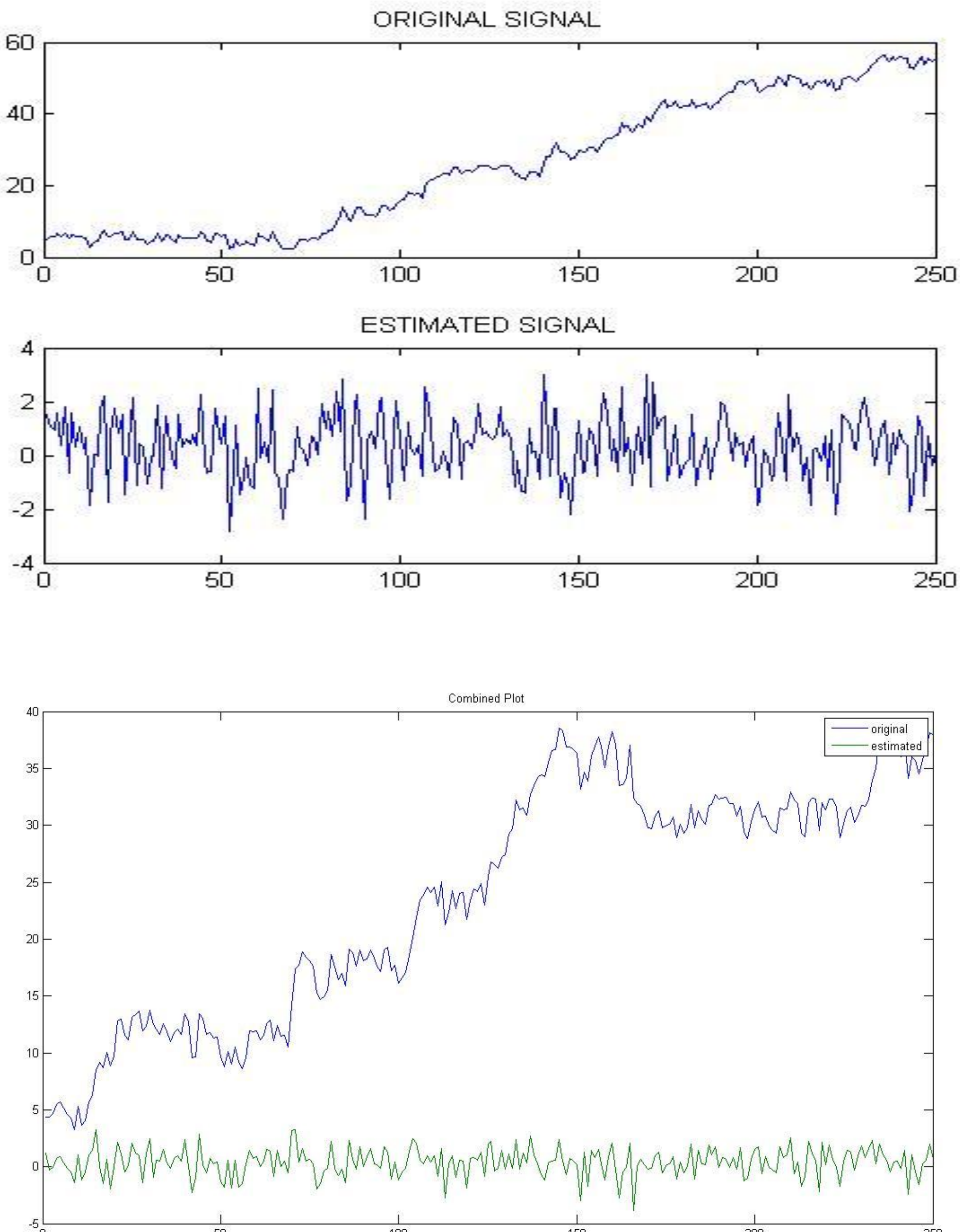


Fig 5.1: Extended Kalman Filter Output

MEAN SQUARE ERROR IN EXTENDED KALMAN FILTER

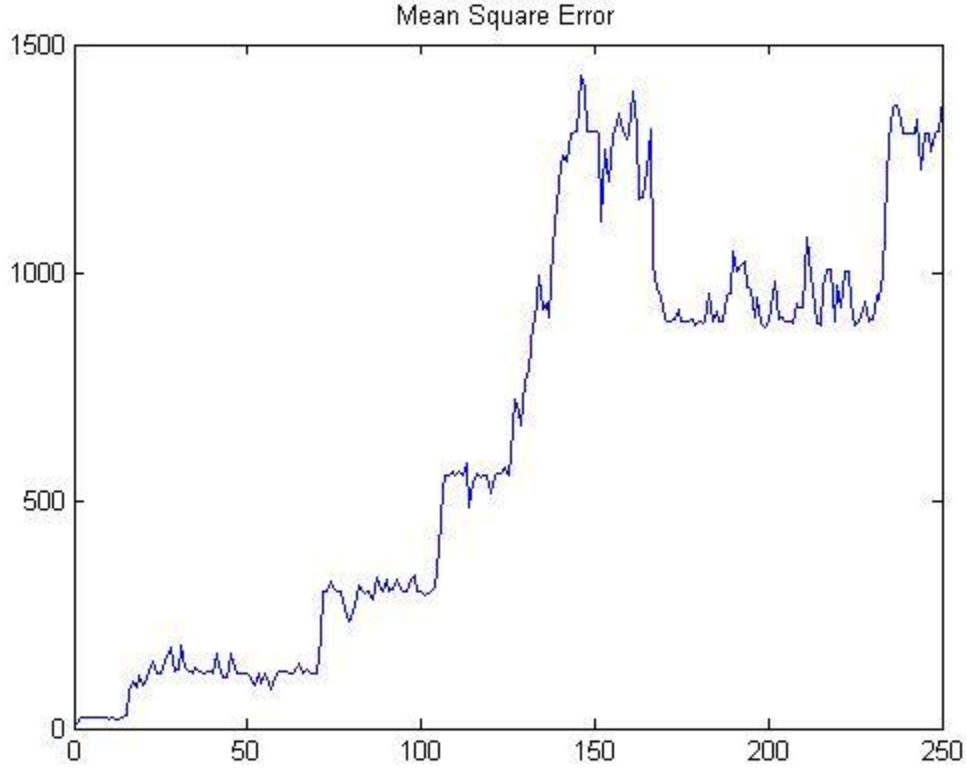
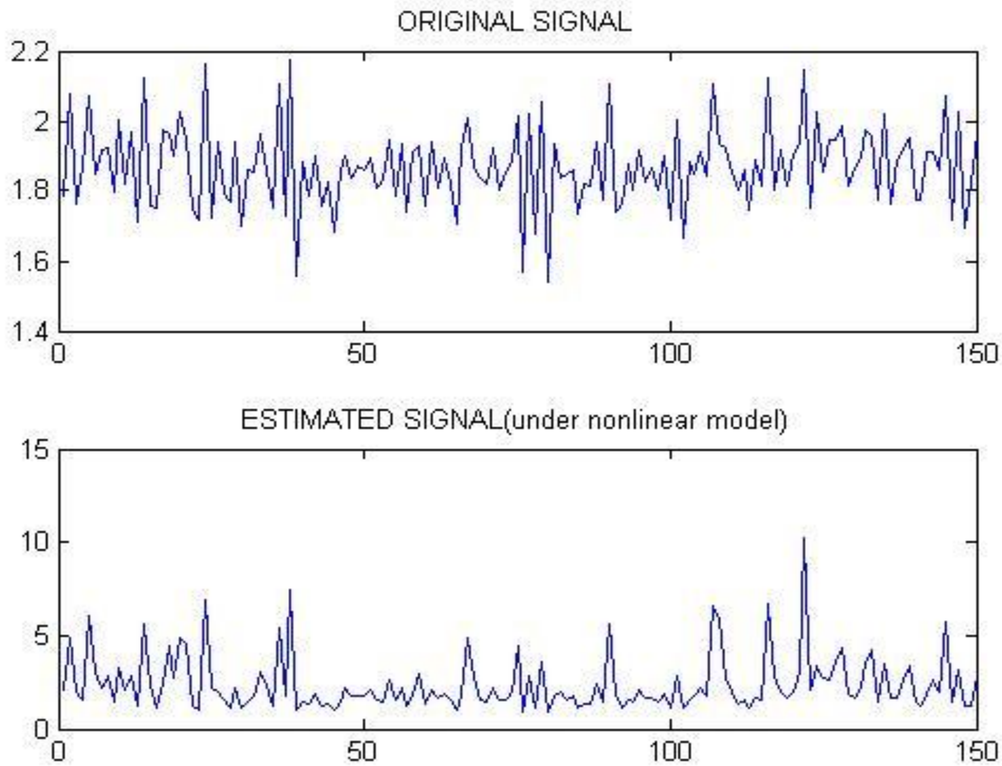


Fig 5.2: Mean Square Error in EKF

UNSCENTED KALMAN FILTER OUTPUT:



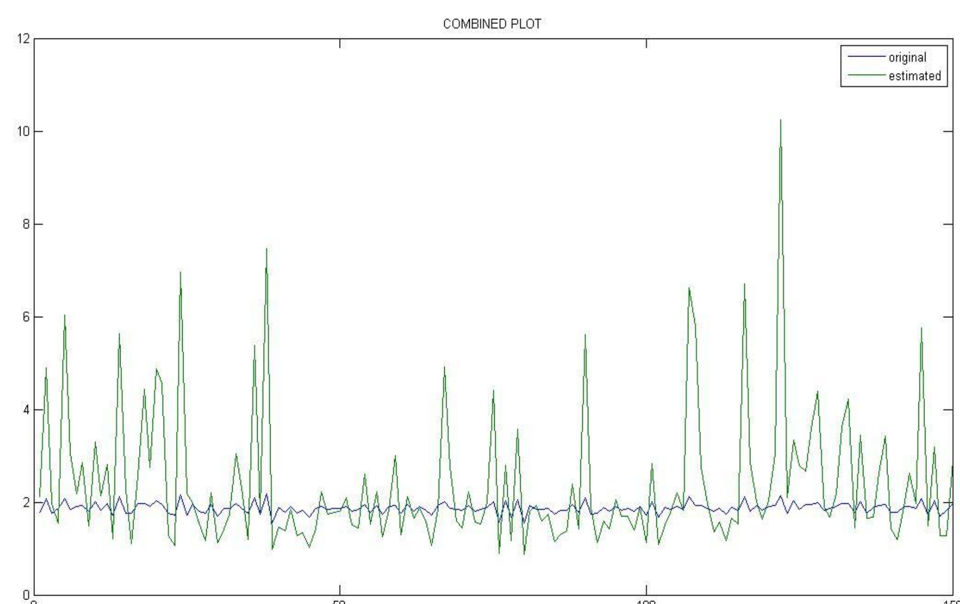


Fig 5.3 : Unscented Kalman Filter Output

FUNDAMENTAL AMPLITUDE ESTIMATION USING EKF:

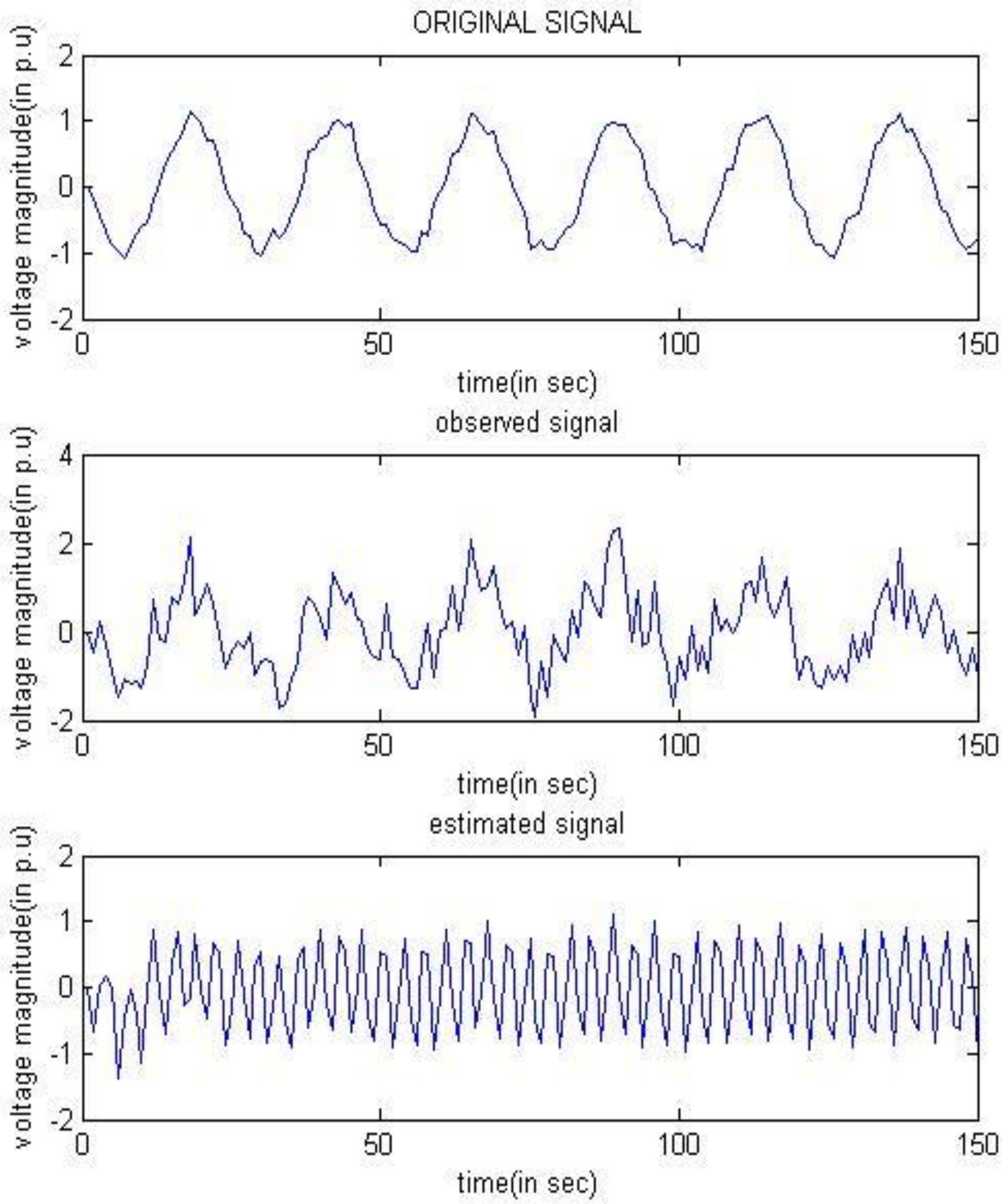
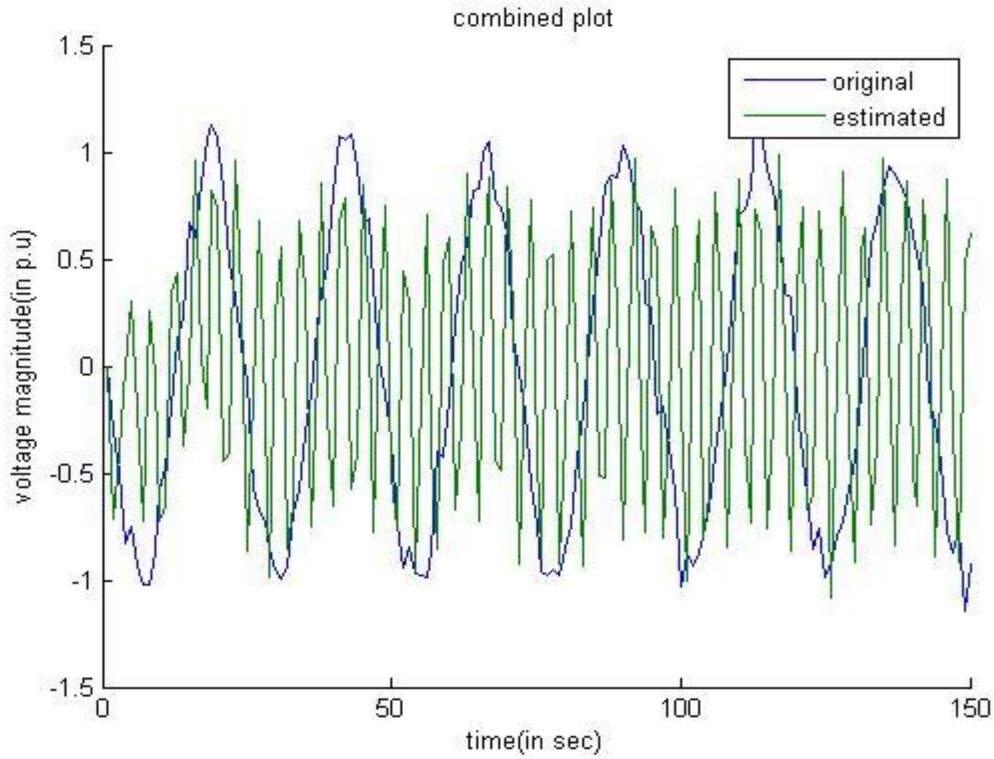


Fig 5.4 : Fundamental amplitude estimation using EKF

FUNDAMENTAL AMPLITUDE AND FREQUENCY ESTIMATION USING EKF:



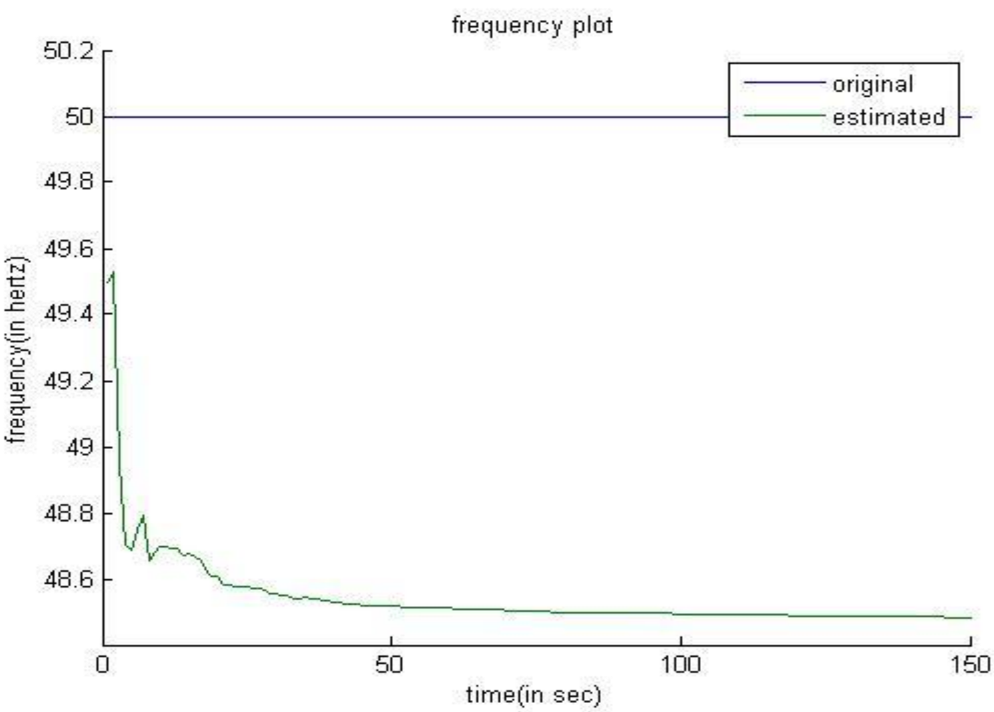


Fig 5.5: Fundamental amplitude and frequency estimation using EKF

3rd HARMONIC AMPLITUDE ESTIMATION (EKF):

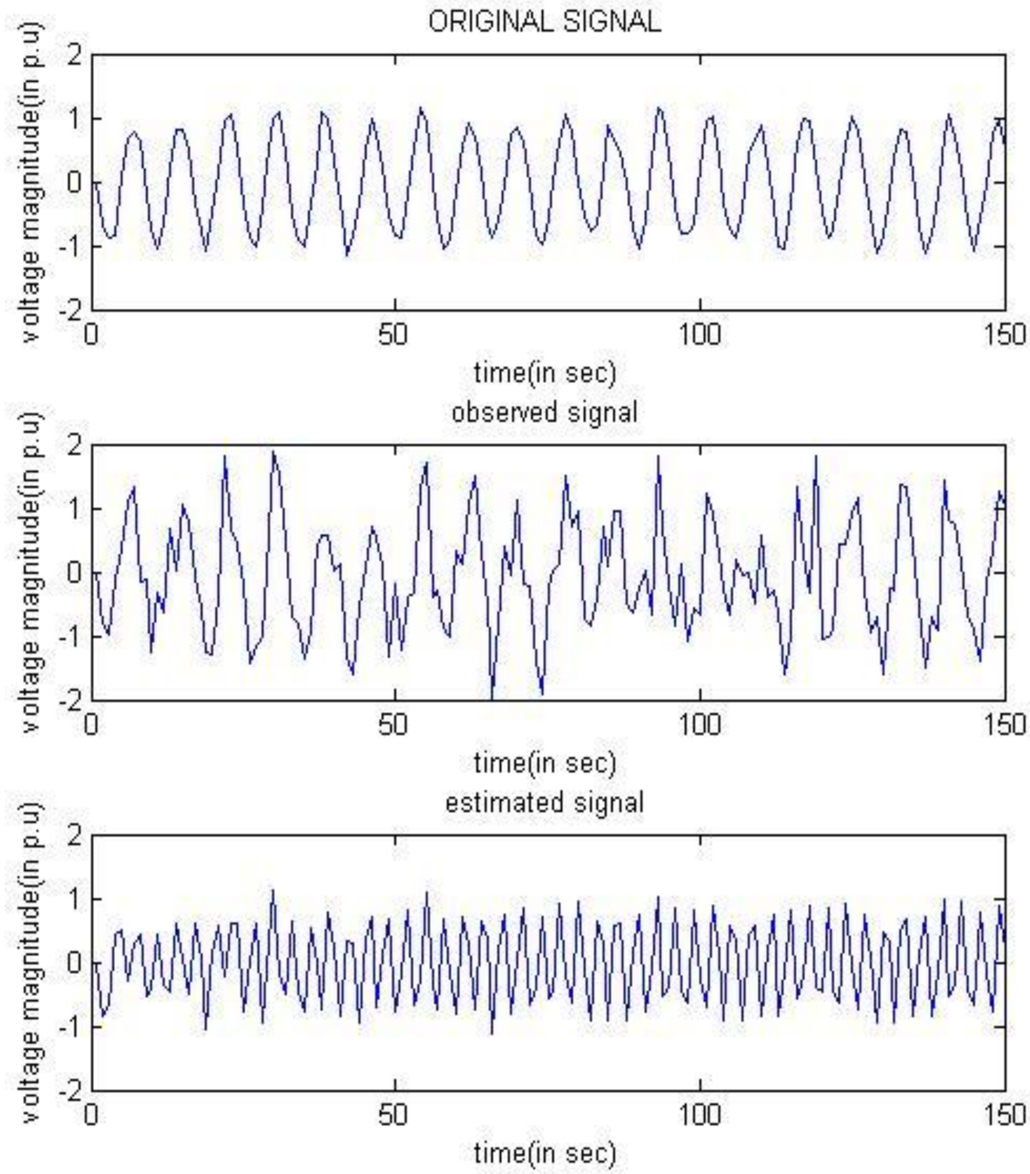
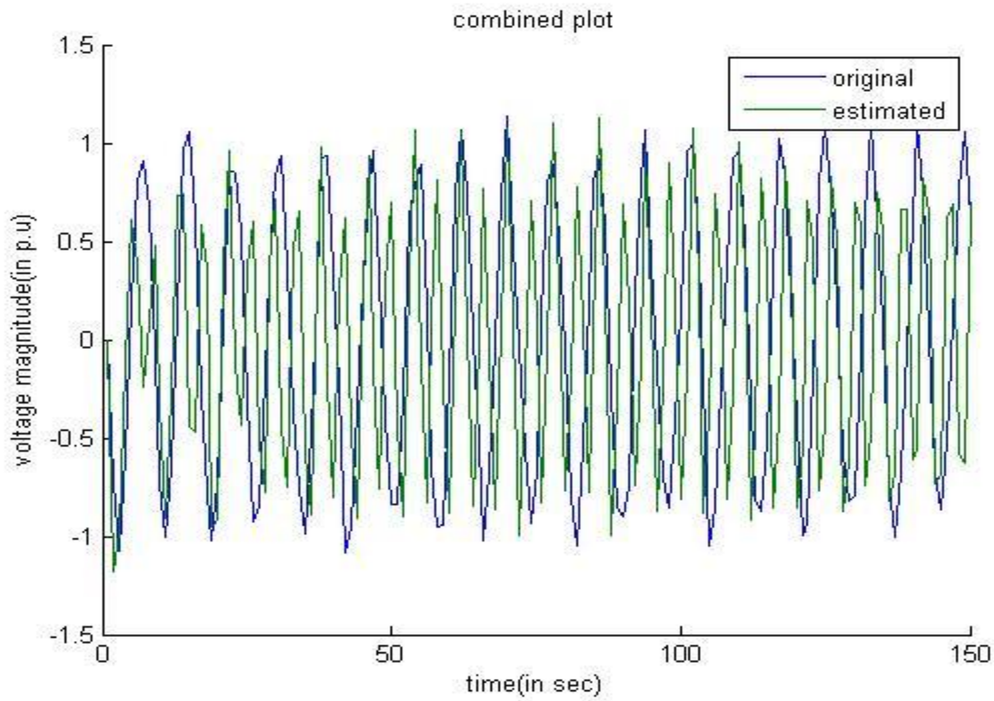


Fig 5.6: 3rd Harmonic amplitude estimation using EKF

3rd HARMONIC AMPLITUDE & FREQUENCY ESTIMATION (EKF):



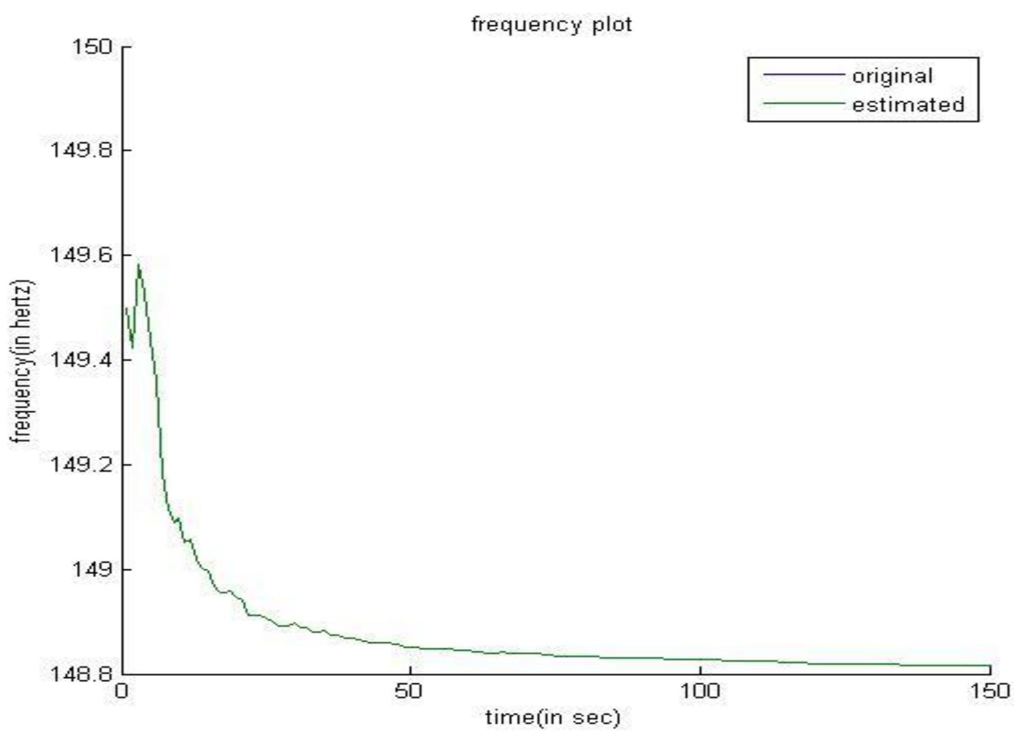


Fig 5.7: 3rd Harmonic amplitude and frequency estimation using EKF

5th HARMONIC APLITUDE ESTIMATION (EKF):

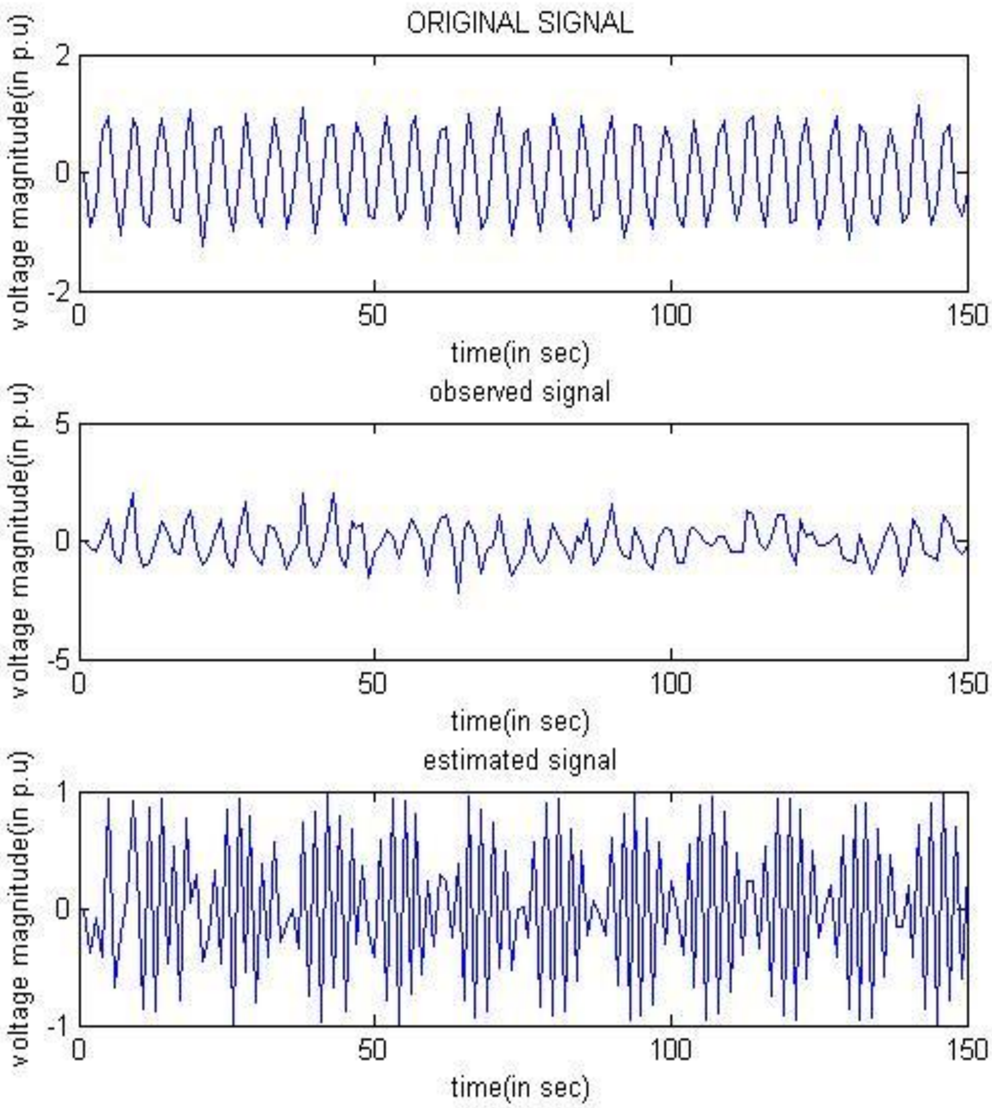


Fig 5.8: 5th Harmonic amplitude estimation using EKF

5^{th} HARMONIC AMPLITUDE & FREQUENCY ESTIMATION (EKF):

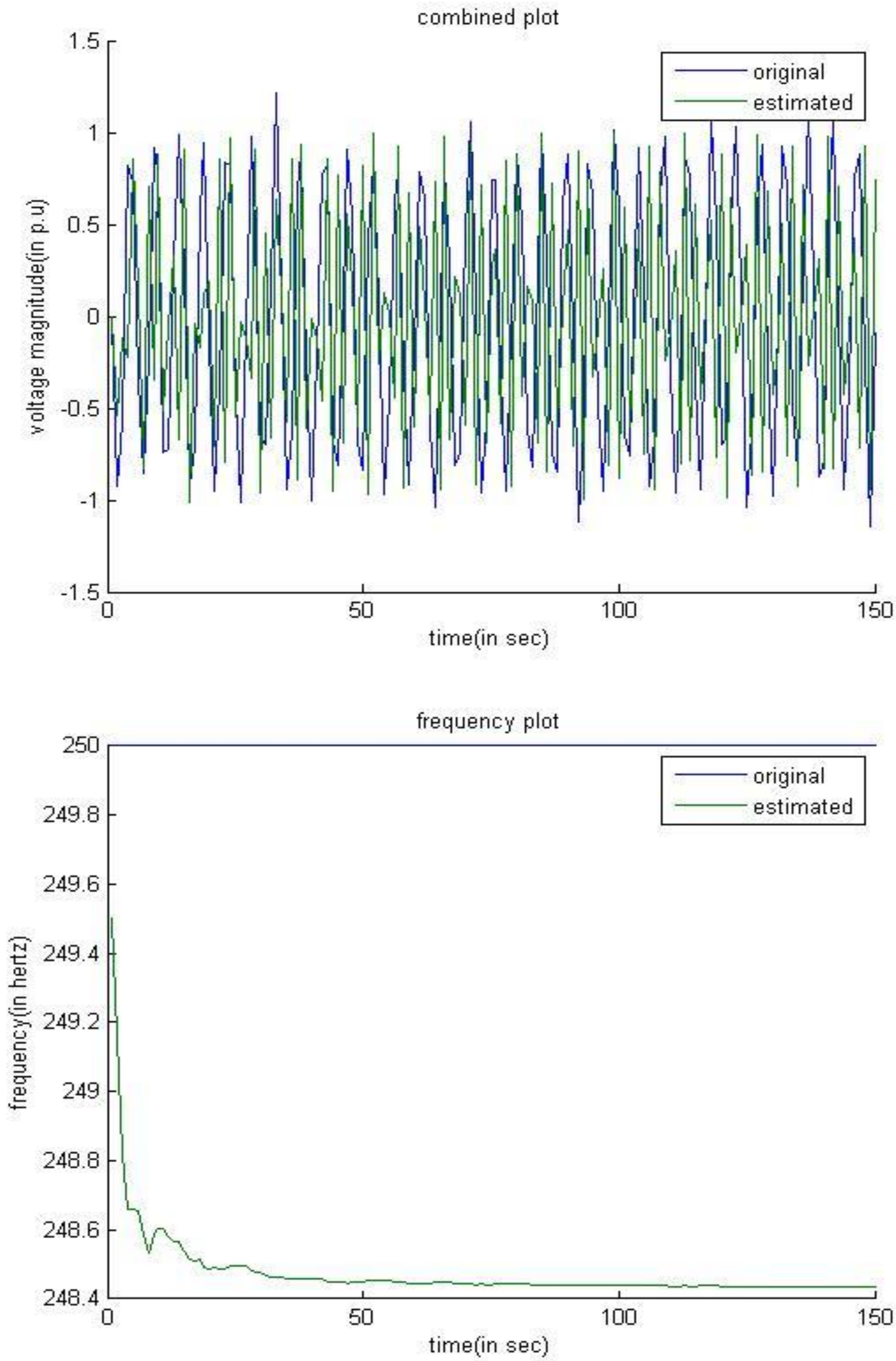


Fig 5.9: 5^{th} Harmonic amplitude and frequency estimation using EKF

7^{th} HARMONIC APLITUDE ESTIMATION (EKF):

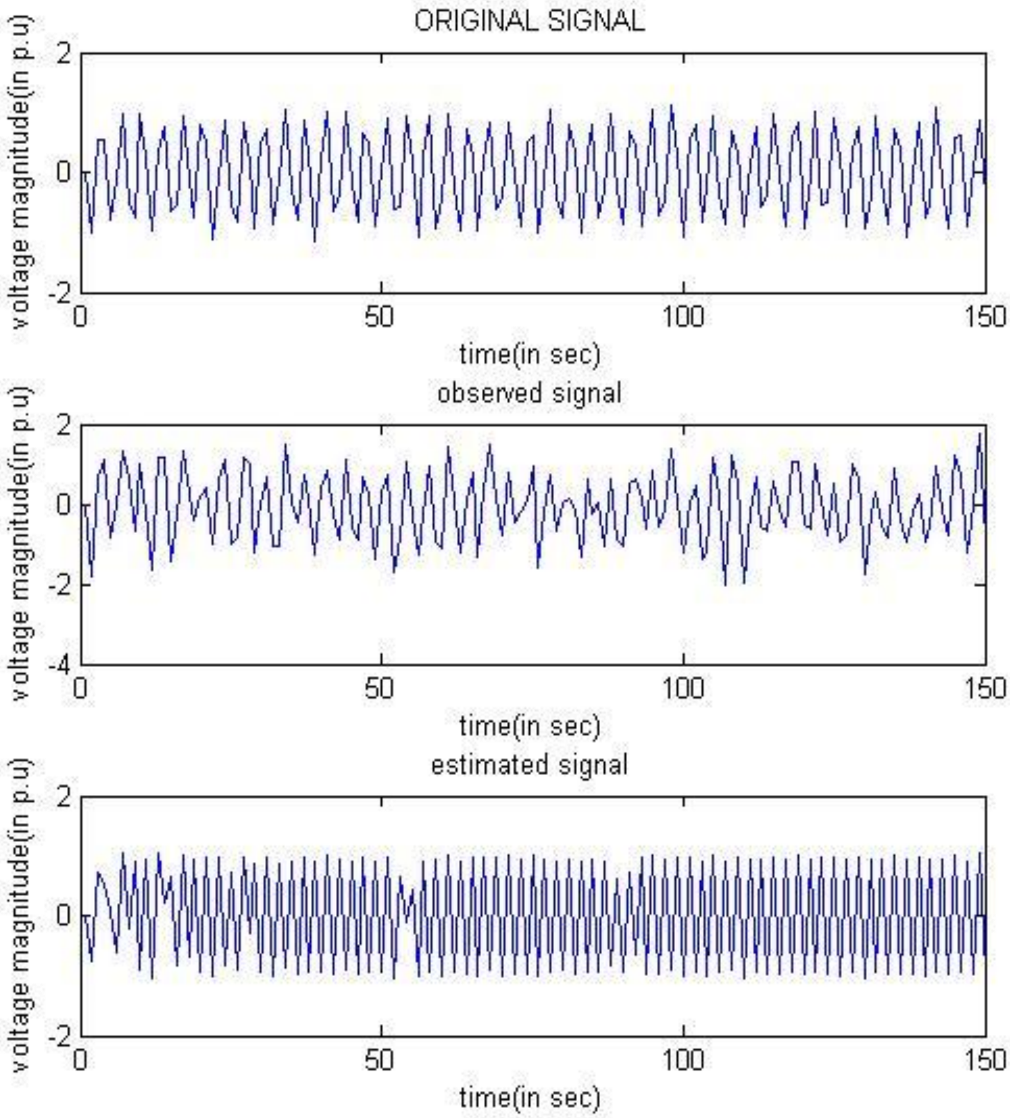
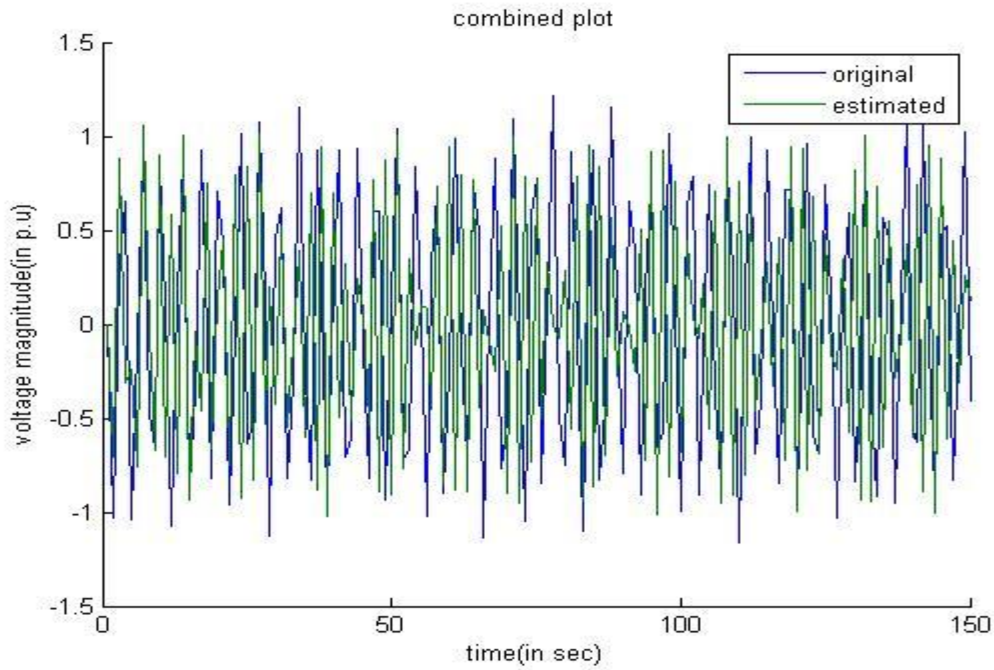


Fig 5.10: 7^{th} Harmonic amplitude estimation using EKF

7^{th} HARMONIC AMPLITUDE & FREQUENCY ESTIMATION (EKF):



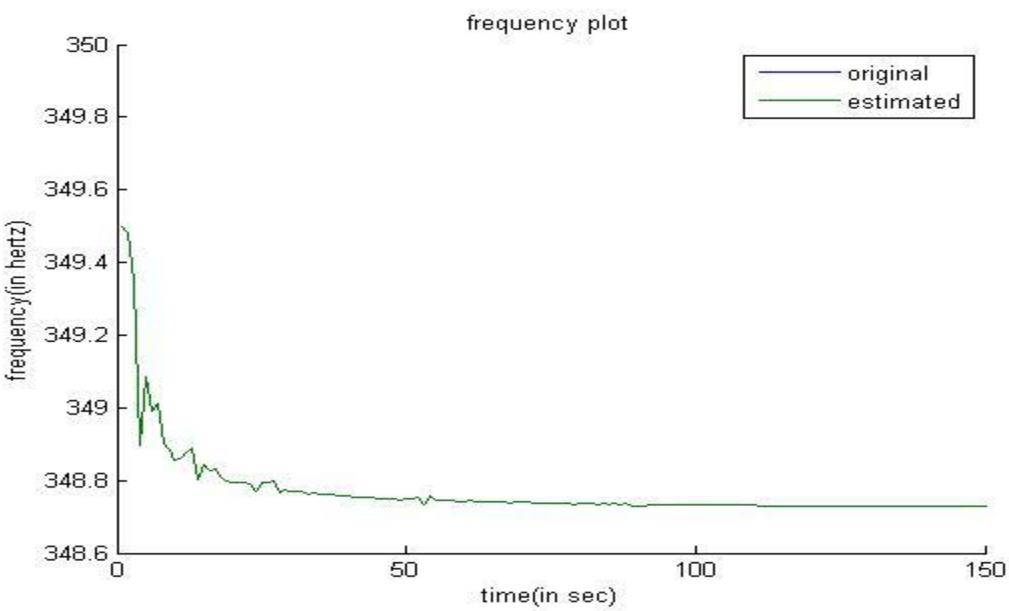


Fig 5.11: 7^{th} Harmonic amplitude and frequency estimation using EKF

CHAPTER – 6

CONCLUSION

CONCLUSION:

Power system efficiency depends upon different aspects and harmonic distortion is one of them. Harmonic content of a power circuit depend upon load. NONLINEAR LOAD and lot of electronic converters in power system are the main cause of harmonics. Performance of communication system channel always related to the harmonic in the transmission line passing side to it. We have mainly two categories of harmonics: Characteristic and Non characteristic harmonic. As per technical aspects is concerned Non characteristic harmonics should not be produced in the power circuit. Characteristic harmonics are integer multiple of fundamental frequency and amplitude is proportional to fundamental component and inversely proportional to order of harmonic. Our necessity is to filter out those harmonics produced and that is why we need a estimator to estimate the parameters of the harmonics.

There are various methods present to estimate and here we are focusing on the famous estimator named kalman filter technique. Kalman filter basically a recursive estimator and uses algorithm based on the least square error. We came to know that simple kalman filter has a limitation that it can work only for linear system and failed in non linear system.

To work with nonlinear system two advanced technique are proposed named extended kalman filter(EKF) and unscented kalman filter (UKF).

As earlier mentioned in Extended kalman filter technique we need to find Jacobeans and which is a very difficult task . In UKF we use a deterministic sampling technique called Unscented transform. Here we need to take minimum number of sigma points around the mean. Then propagating these sigma points through the nonlinear functions, mean and covariance of the estimate are recovered.in UKF we can get more accurate true mean and variance. In addition we do not need to find Jacobeans as in EKF.

Our project mainly based on the comparison of these two advanced kalman filter: EKF and UKF.

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